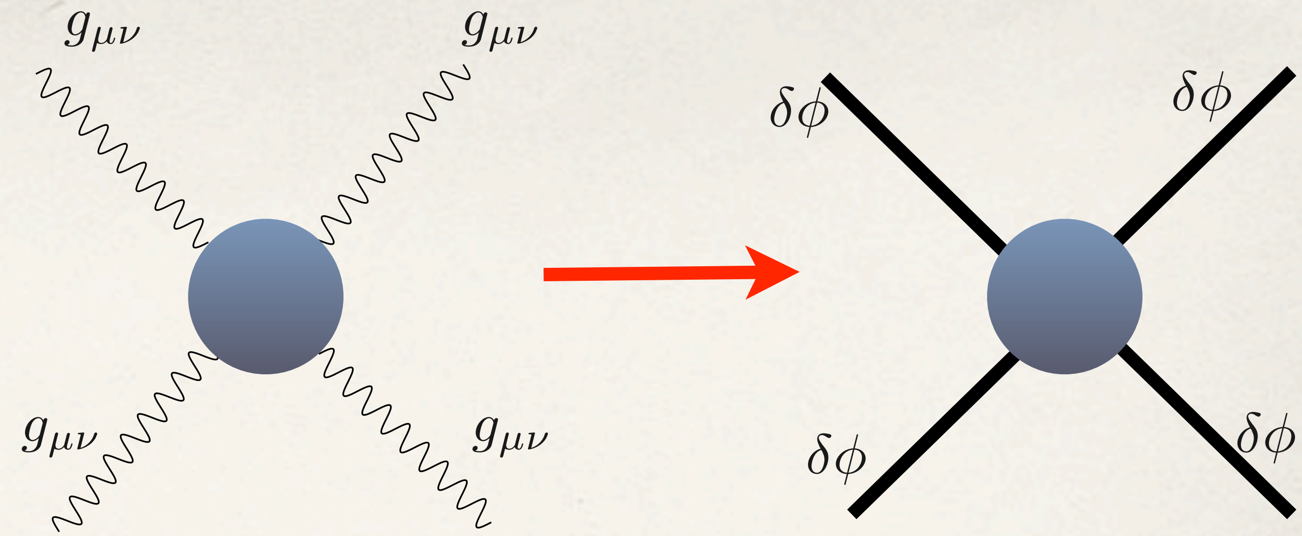
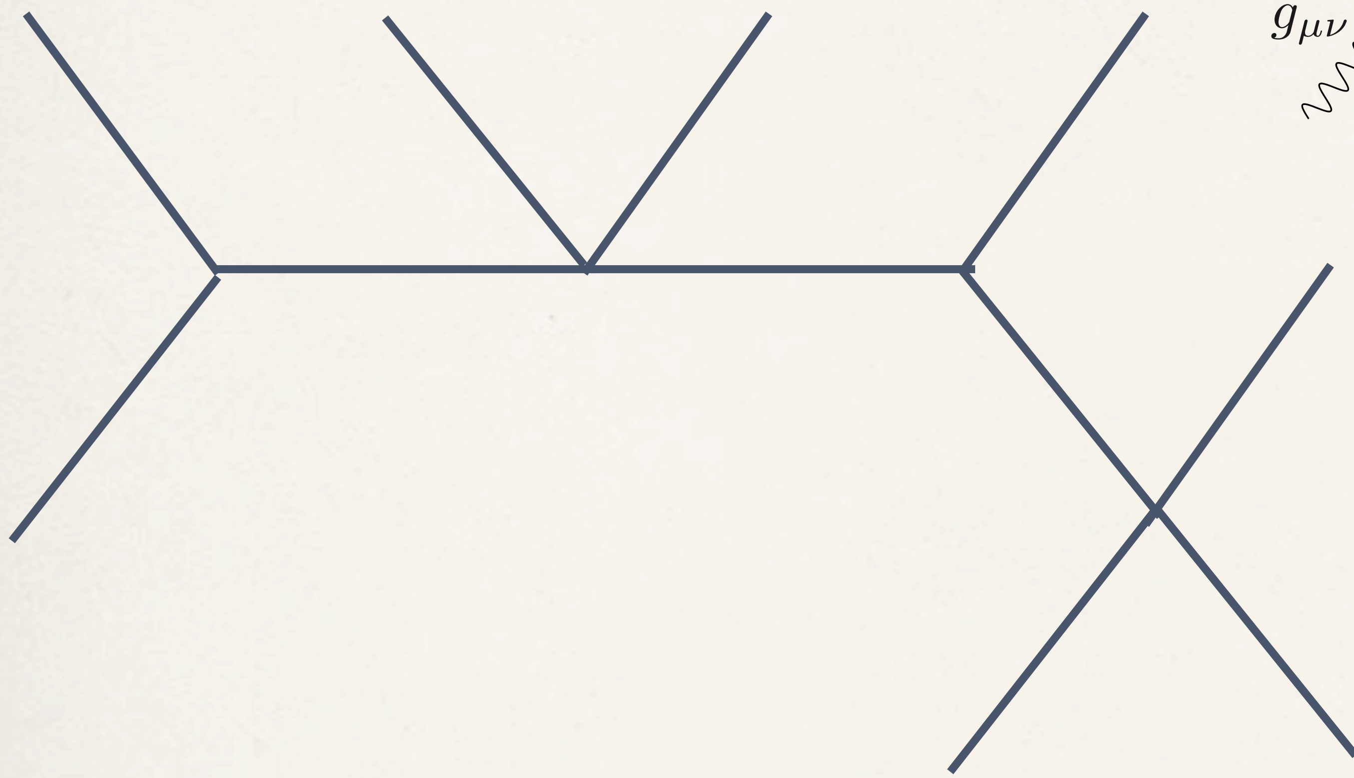
$$\langle \mathcal{R}_{\mathbf{k}_1} \cdots \mathcal{R}_{\mathbf{k}_N} \rangle$$



# Beyond the Bispectrum: N-point Functions for large N


Louis Leblond

Perimeter Institute



**arXiv:1010.4565, with Enrico Pajer**

Majority of theoretical studies have focused on the bispectrum

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta^3 \left(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}\right) B(k_i)$$

One of the theme of this NG workshop is lets go beyond local bispectra

e.g

$\tau_{NL}$

Scale dependence

Here lets go way beyond.....
Report a model where we have a good handle on N-point functions for N of order 10 to 25.

$$\langle \mathcal{R}_{\mathbf{k}_1} \cdots \mathcal{R}_{\mathbf{k}_N} \rangle = (2\pi)^3 \delta^3 \left( \sum_i^N \mathbf{k_i} \right) B_N(\mathbf{k_i})$$

This is a function of 3N-6 variables

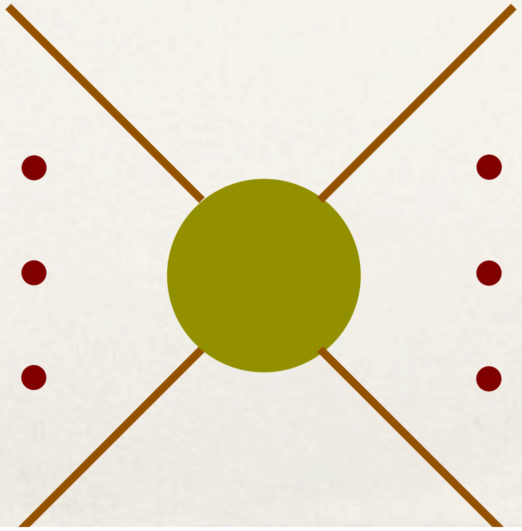
This is a nice toy model to learn about the structure of higher point correlation functions.

# The challenge for theorists

- The main bottleneck for predicting the standard model background at the LHC is coming from the theorist's ability to compute higher point function in gauge theories.

- Usually quite hard to calculate beyond the power spectrum in theories of inflation mostly because of gravity.

- Many bispectra are known, some trispectra (a lot of work) are out there and essentially nothing beyond.

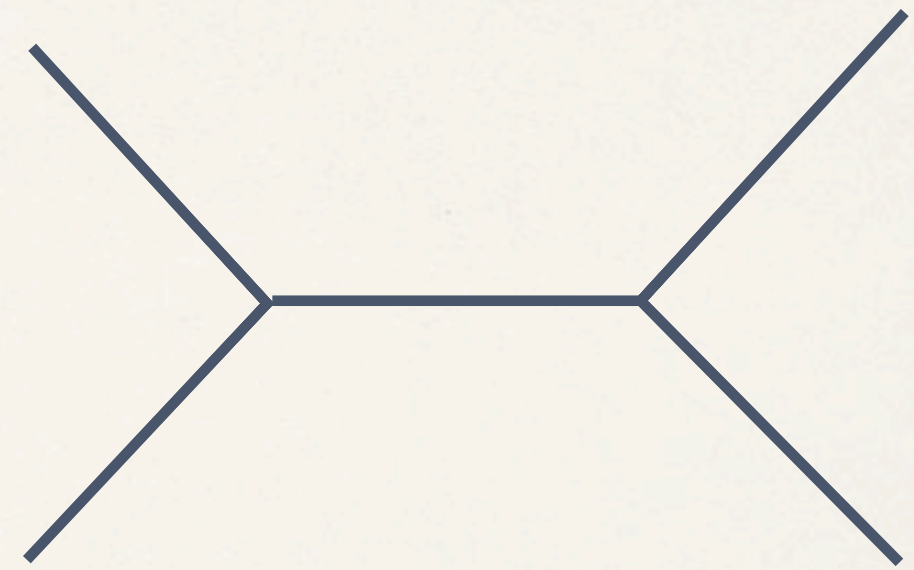


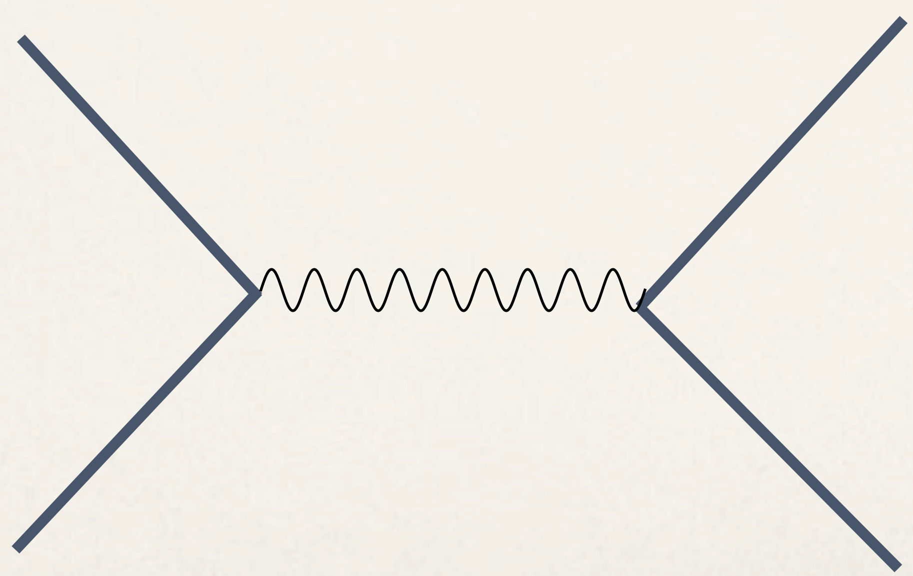
By decoupling gravity in a particular model of inflation we will obtain all correlation functions at tree level up to N of order 10, 12 or even ~25 !

Leblond, NG2011, MCTP

# Slow-roll Trispectrum

$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \mathcal{R}_{\mathbf{k}_4} \rangle$





Seery, Lidsey, Sloth
Arroja, Koyama
Seery, Sloth, Vernizzi

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$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \mathcal{R}_{\mathbf{k}_4} \rangle$

$$
\begin{aligned}
S_4 = \int a^3 \Big( & -\frac{1}{24} V_{,\alpha\beta\gamma\delta} \delta\phi^\alpha \delta\phi^\beta \delta\phi^\gamma \delta\phi^\delta + \frac{1}{2a^4} \partial_{(i}\beta_{2j)} \partial_i \beta_{2j} \\
& + \frac{1}{2a^4} \partial_j \vartheta_1 \partial_j \delta\phi^\alpha \partial_m \vartheta_1 \partial_m \delta\phi_\alpha - \frac{1}{a^2} \delta\dot{\phi}^\alpha (\partial_j \vartheta_2 + \beta_{2j}) \partial_j \delta\phi_\alpha \\
& + (\alpha_1^2 \alpha_2 - \frac{1}{2}\alpha_2^2)(-6H^2 + \dot{\phi}^\alpha \dot{\phi}_\alpha) + \frac{\alpha_1}{2} \Big[ -\frac{1}{3} V_{,\alpha\beta\gamma} \delta\phi^\alpha \delta\phi^\beta \delta\phi^\gamma - 2\alpha_1^2 V_{,\alpha} \delta\phi^\alpha \\
& \quad + \alpha_1 \Big( -\frac{1}{a^2} \partial_i \delta\phi^\alpha \partial_i \delta\phi_\alpha - V_{,\alpha\beta} \delta\phi^\alpha \delta\phi^\beta \Big) \\
& \quad - \frac{2}{a^4} \partial_i \partial_j \vartheta_2 \partial_i \partial_j \vartheta_1 + \frac{2}{a^4} \partial^2 \vartheta_2 \partial^2 \vartheta_1 - \frac{2}{a^4} \partial_i \beta_{2j} \partial_i \partial_j \vartheta_1 \\
& \quad + \frac{2}{a^2} \dot{\phi}^\alpha (\partial_j \vartheta_2 + \beta_{2j}) \partial_j \delta\phi_\alpha + \frac{2}{a^2} \delta\dot{\phi}^\alpha \partial_j \vartheta_1 \partial_j \delta\phi_\alpha \Big] \Big). \qquad (36)
\end{aligned}
$$

Seery, Lidsey, Sloth
Arroja, Koyama
Seery, Sloth, Vernizzi

from astro-ph/0610210

# Slow-roll Trispectrum

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from astro-ph/0610210

Seery, Lidsey, Sloth
Arroja, Koyama
Seery, Sloth, Vernizzi

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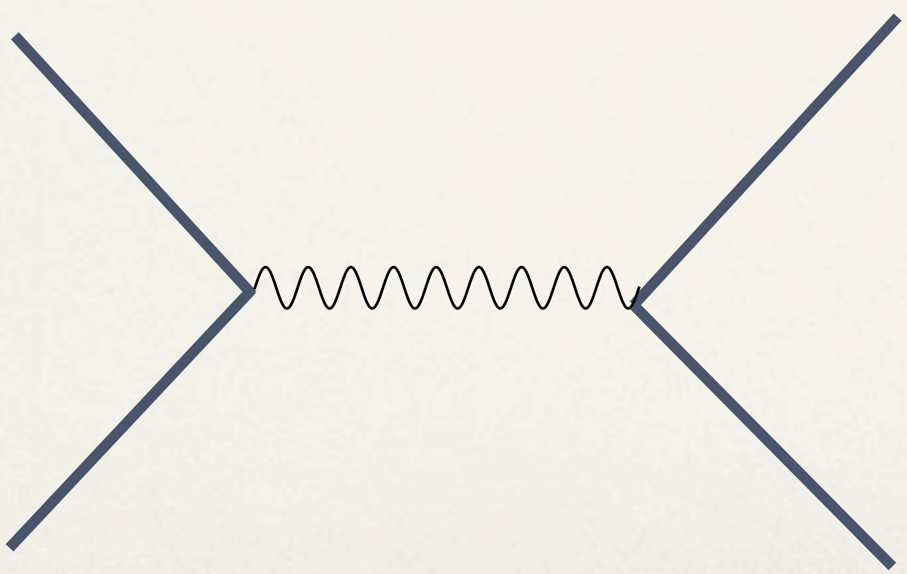
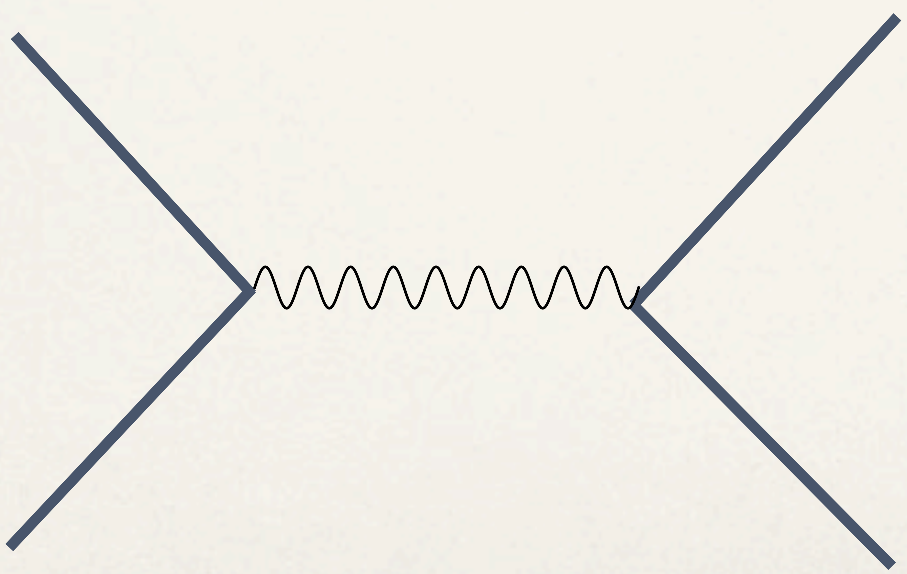


Seery, Lidsey, Sloth
Arroja, Koyama
Seery, Sloth, Vernizzi

# Slow-roll Trispectrum

$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \mathcal{R}_{\mathbf{k}_4} \rangle$

$$\langle \varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\varphi_{\mathbf{k}_4}\rangle_*^{\mathrm{GE}} = (2\pi)^3\delta(\sum_a \mathbf{k}_a)\frac{4H_*^6}{\prod_a(2k_a^3)}$$
$$\times \sum_s \Big[\frac{1}{k_{12}^3}\epsilon_{ij}^s(\mathbf{k}_{12})\epsilon_{lm}^s(\mathbf{k}_{34})k_1^i k_2^j k_3^l k_4^m \cdot (\mathcal{I}_{1234} + \mathcal{I}_{3412})$$
$$+ \frac{1}{k_{13}^3}\epsilon_{ij}^s(\mathbf{k}_{13})\epsilon_{lm}^s(\mathbf{k}_{24})k_1^i k_3^j k_2^l k_4^m \cdot (\mathcal{I}_{1324} + \mathcal{I}_{2413})$$
$$+ \frac{1}{k_{14}^3}\epsilon_{ij}^s(\mathbf{k}_{14})\epsilon_{lm}^s(\mathbf{k}_{23})k_1^i k_4^j k_2^l k_3^m \cdot (\mathcal{I}_{1423} + \mathcal{I}_{2314})\Big] \,. \quad (2.25)$$



from 0811.3934

Seery, Lidsey, Sloth
Arroja, Koyama
Seery, Sloth, Vernizzi

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$$\mathcal{I}_{1234} + \mathcal{I}_{3412} = \frac{k_1 + k_2}{a_{34}^2} \left[ \frac{1}{2}(a_{34} + k_{12})(a_{34}^2 - 2b_{34}) + k_{12}^2(k_3 + k_4) \right] + (1,2 \leftrightarrow 3,4)$$
$$+ \frac{k_1 k_2}{k_t} \left[ \frac{b_{34}}{a_{34}} - k_{12} + \frac{k_{12}}{a_{12}} \left( k_3 k_4 - k_{12} \frac{b_{34}}{a_{34}} \right) \left( \frac{1}{k_t} + \frac{1}{a_{12}} \right) \right] + (1,2 \leftrightarrow 3,4)$$
$$- \frac{k_{12}}{a_{12} a_{34} k_t} \left[ b_{12} b_{34} + 2k_{12}^2 (\prod_a k_a) \left( \frac{1}{k_t^2} + \frac{1}{a_{12} a_{34}} + \frac{k_{12}}{k_t a_{12} a_{34}} \right) \right] , \quad (2.26)$$

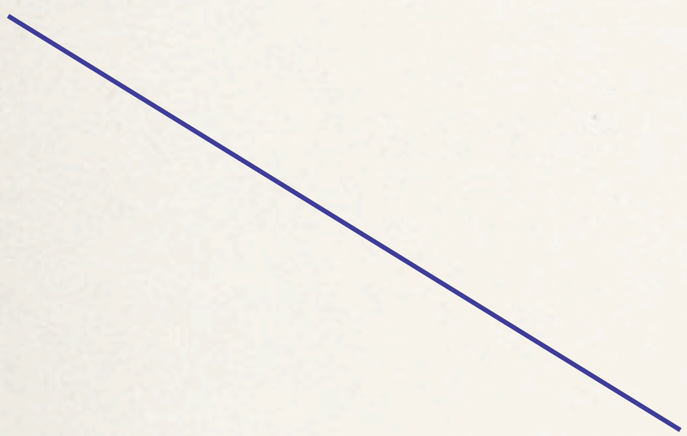
where we have used $k_{12} = k_{34}$ and we have defined

$$a_{ab} \equiv k_a + k_b + k_{ab} , \qquad b_{ab} \equiv (k_a + k_b) k_{ab} + k_a k_b . \quad (2.27)$$

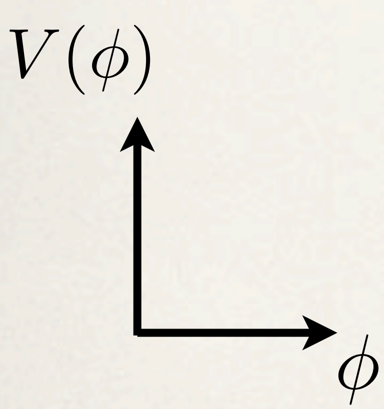
Seery, Lidsey, Sloth
Arroja, Koyama
Seery, Sloth, Vernizzi

from 0811.3934

# Resonant Inflationary Models



Slow-roll potential




$$V(\phi) = V_{\rm sr}(\phi) + \Lambda^4 \cos(\phi/f)$$

Amplitude

b<< 1

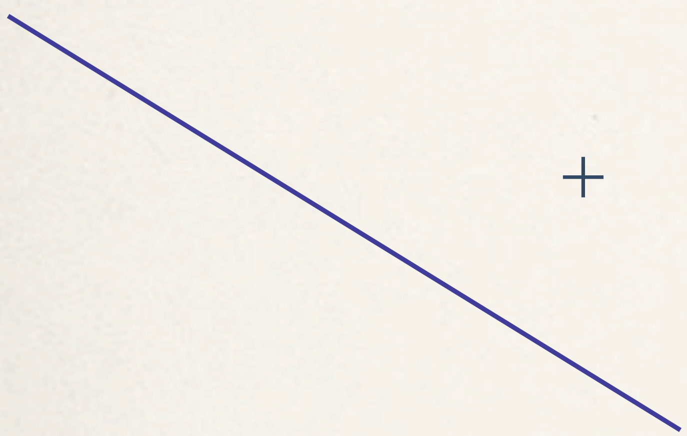
Frequency

α >> 1

Chen, Easther, Lim
Flauger, Pajer

Barnaby, Peloso

# Resonant Inflationary Models



+

Slow-roll potential

Modulation

Amplitude

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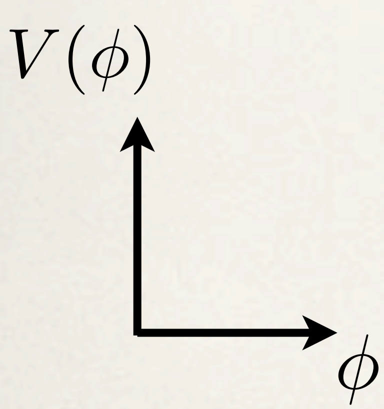
Frequency

α >> 1

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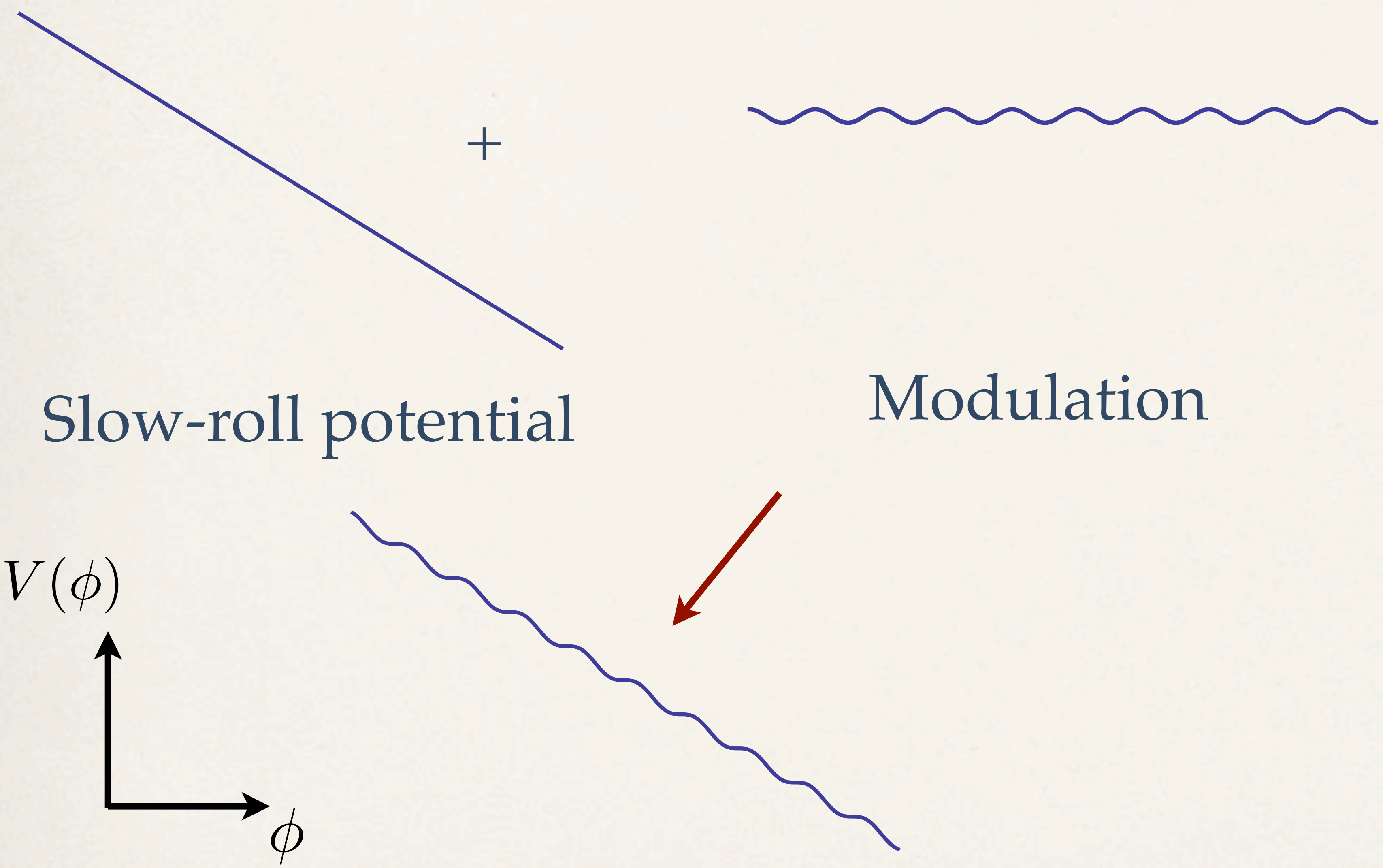


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Chen, Easther, Lim
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# Results

$$\langle \mathcal{R}_{\mathbf{k}_1} \cdots \mathcal{R}_{\mathbf{k}_N} \rangle^{\text{single vertex}} = (2\pi)^3 \delta^3 \left( \sum_i^N \mathbf{k}_i \right) A_N B_N(k_i)$$

amplitude

$$A_N \equiv (-)^N \frac{3b\sqrt{2\pi}}{2} \alpha^{2N-9/2} (2\pi^2 \Delta_R^2)^{N-1}$$

shape

$$B_N(k_i) \equiv \frac{1}{K^{N-3} \prod_i k_i^2} \sin\left( \frac{\phi_K}{f} \right)$$

leading

$$K = \sum_i k_i$$

Signal oscillates

$$\phi_K = \phi_* - \sqrt{2\epsilon_*} \ln K/k_*$$

Essentially no overlap to most other shape.

only a function of N variable (norms) and not 3N-6
as one would naively expect.

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leading + 1/α

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Essentially no overlap to most other shape.

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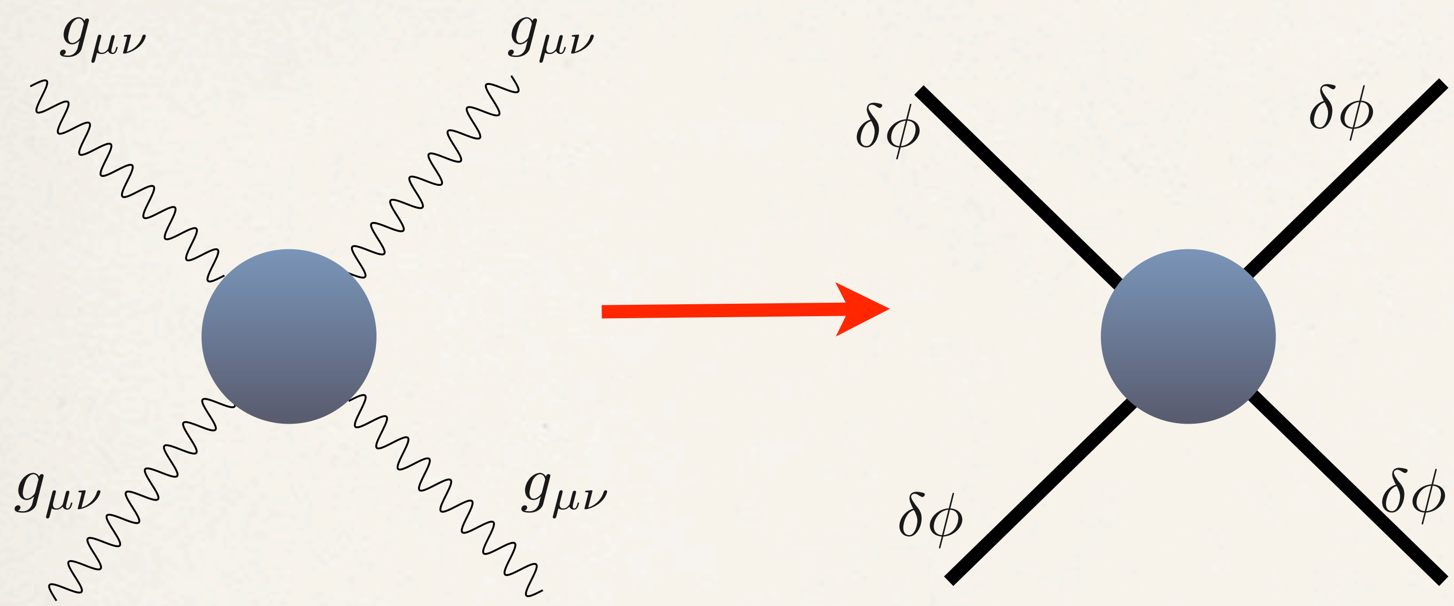
# Decoupling

Claim: All single field models of inflation with parametrically large NG admit a decoupling limit in which




Self-interactions of the field ϕ dominate over all gravitational interactions.

$$g_{\mu\nu} \to \quad h_{ij} \; + \; \delta\phi$$

Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore

For Resonant

$$V(\phi) = 3\alpha b f^2 H^2 \cos(\phi/f)$$

$$\mathcal{R} = -\frac{H}{\dot{\phi}}\delta\phi$$

Vertices $$\frac{V^{(N)}}{N!}\delta\phi(x)^N$$

$$\langle \mathcal{R}_{\mathbf{k}_1} \cdots \mathcal{R}_{\mathbf{k}_N} \rangle^{\text{single vertex}}$$

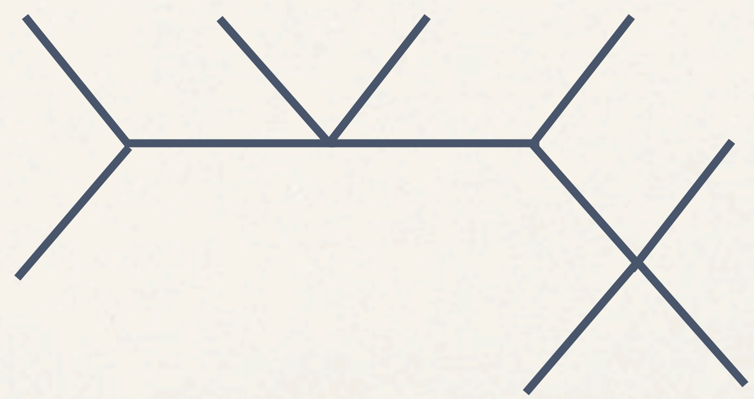
$$\langle \mathcal{R}^5 \rangle \rightarrow$$

$$B_N(k_i) \equiv \frac{1}{K^{N-3}\prod_i k_i^2}\left[\sin\left(\frac{\phi_K}{f}\right) - \frac{1}{\alpha}\cos\left(\frac{\phi_K}{f}\right)\sum_{j,i}\frac{k_i}{k_j} + \mathcal{O}(\alpha^{-2})\right]$$

Limit of validity

Gravity kicks in here

- Neglecting Gravity
- Multi-vertex diagrams.



- Special choice of momenta.

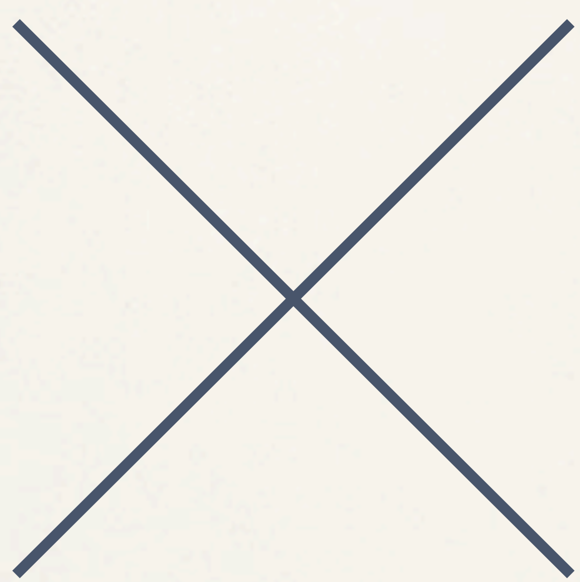
Leblond, NG2011, MCTP

Any single field model of inflation with large NG decouples

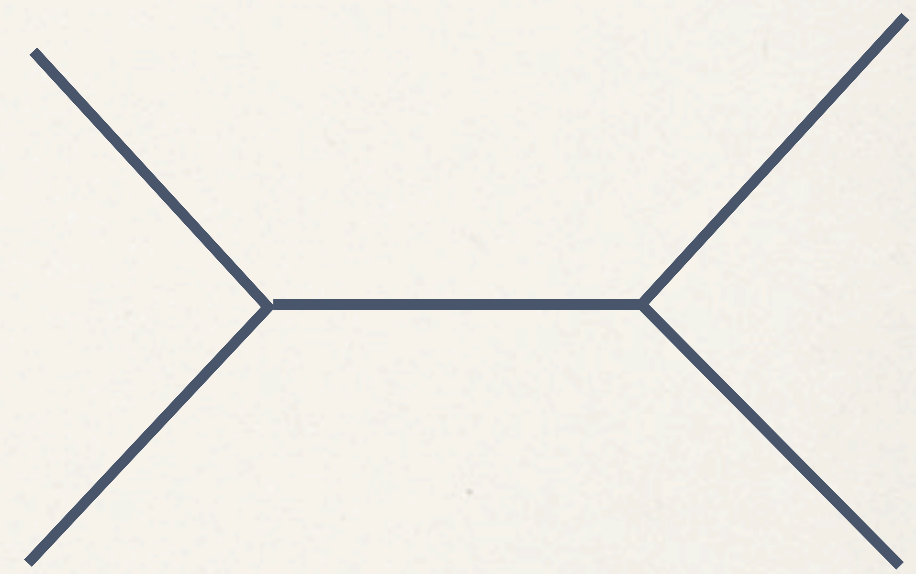
e.g. DBI inflation

But for most of the models, there are

Many contact terms



$\sim$



DBI

Chen, Huang, Shiu
Arroja, Mizuno, Koyama, Tanaka
L.L. Shandera

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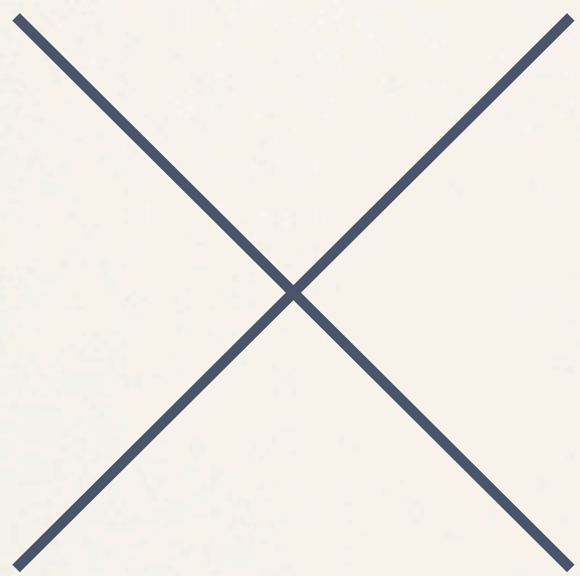

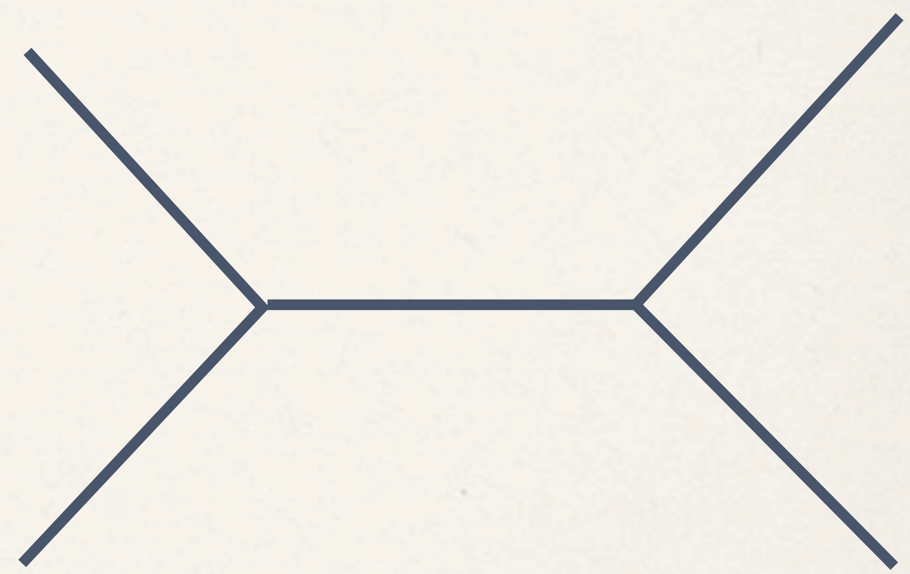

Chen, Huang, Shiu
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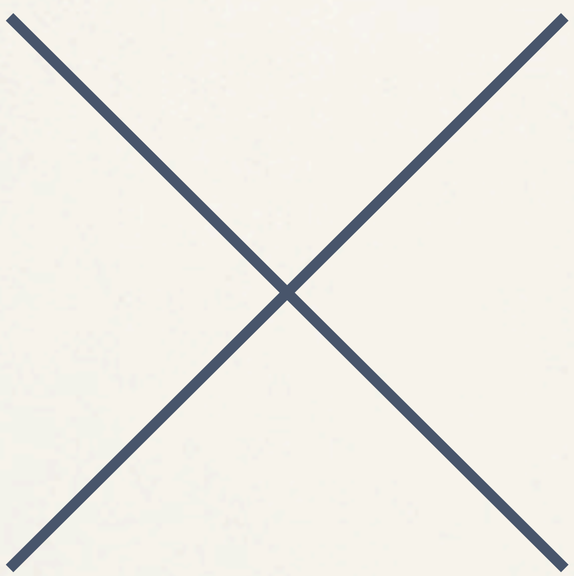

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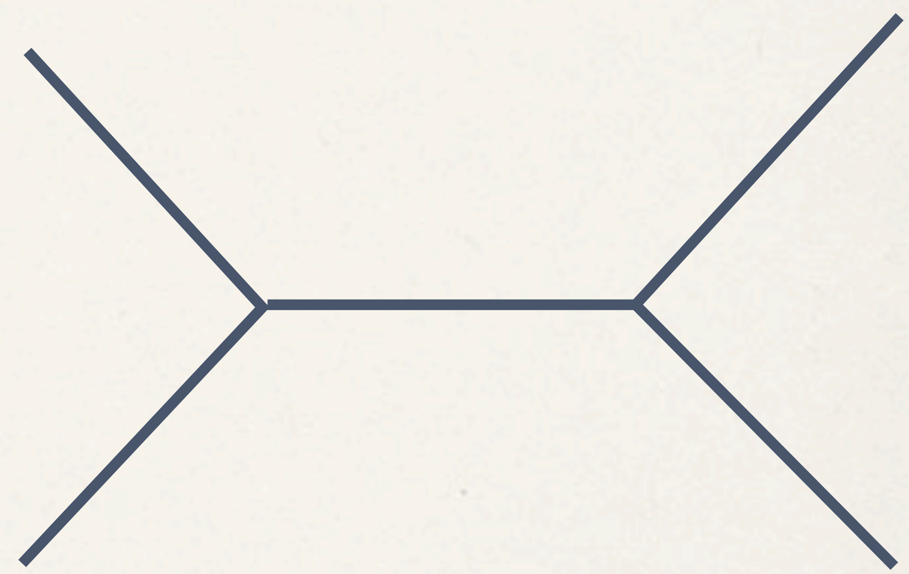


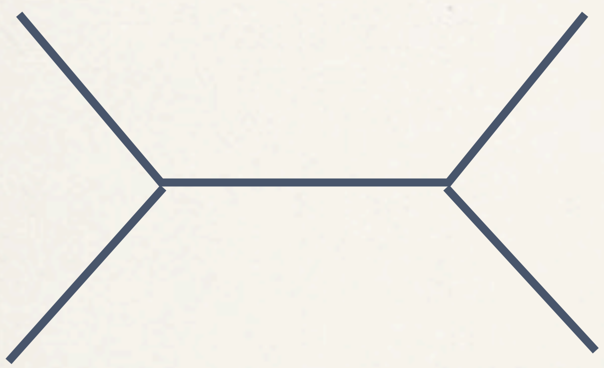
Resonant

Chen, Huang, Shiu
Arroja, Mizuno, Koyama, Tanaka
L.L. Shandera

1st problem
and most serious

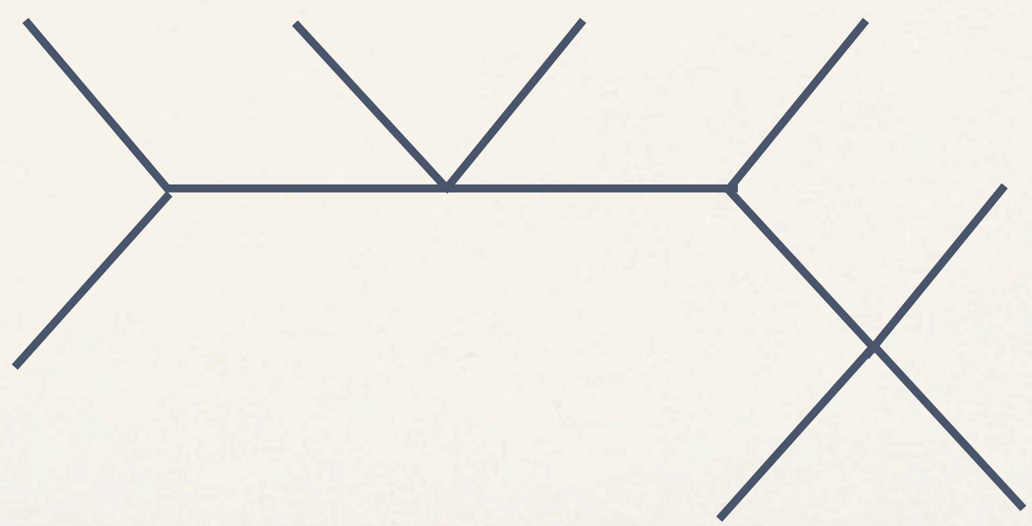
# Multi-vertex diagrams

Starting with 4-pt, there exists multi-vertex diagrams.

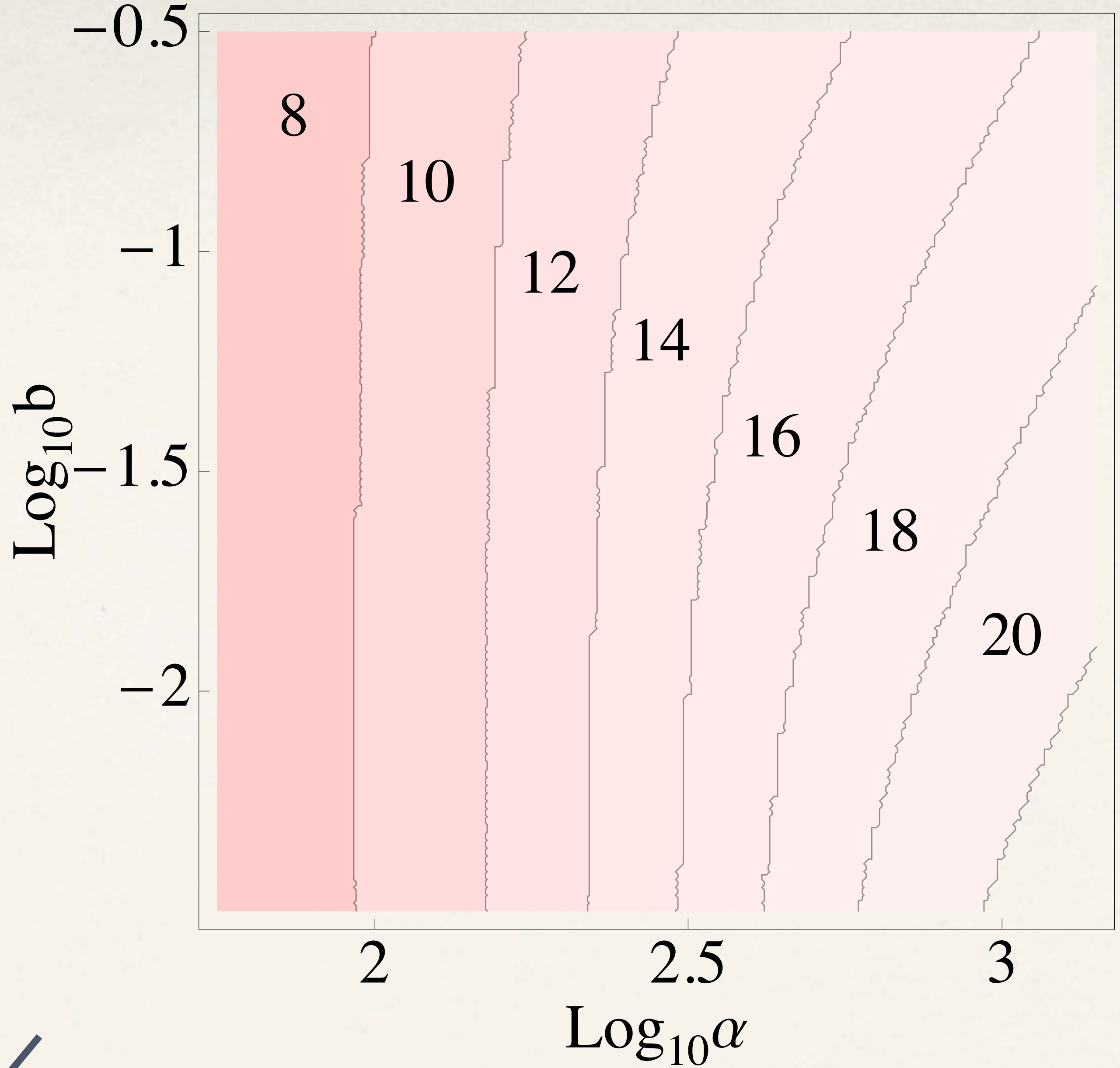


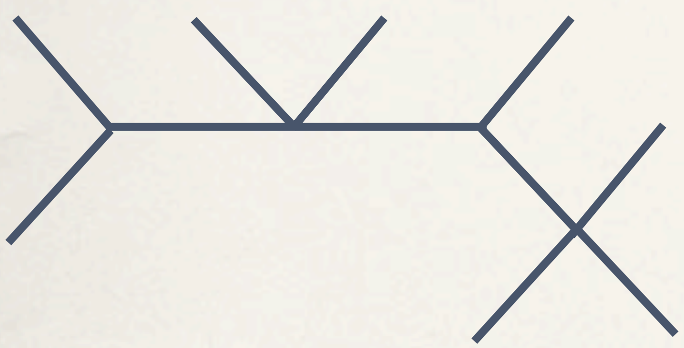
they are subleading (more b and suppressed in α)
but there is many of them

really a lot of terms



For α~ 100 and b~0.1 we estimate that the SUM of all multi-vertex contribute less than 20% for N< 10



Maximum N for which
our formula can be trusted

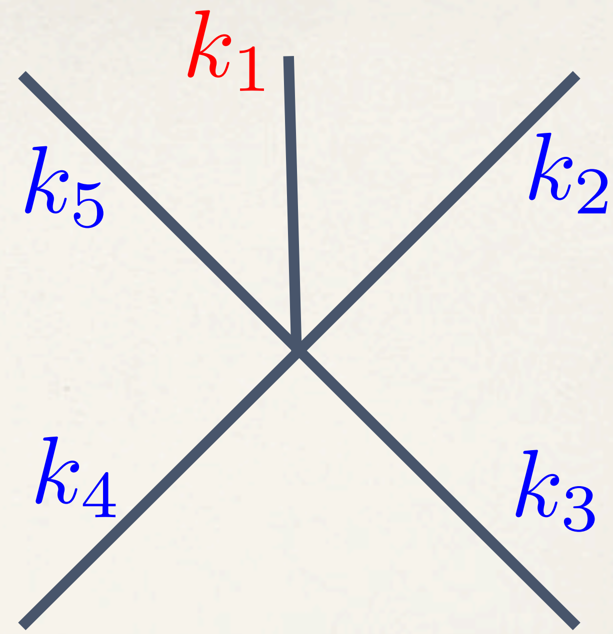
Above this N, the sum over all subleading diagrams becomes of order 20% of the leading answer

Leblond, NG2011, MCTP

2nd Problem

# Squeezed and collinear Limits

$$\lim_{k_1 \to 0}$$




Squeezed (soft)

Correlation Functions can be enhanced in some small region of momentum spaces (near poles)

$$B_N(k_i) \equiv \frac{1}{K^{N-3}\prod_i k_i^2} \sin\left(\frac{\phi_K}{f}\right)$$

$$\lim_{k_1 \to 0} \langle \mathcal{R}^5 \rangle_{\alpha-\text{leading}} \propto \frac{1}{k_1^2 k^{10}}$$

$$\lim_{k_1 \to 0} \langle \mathcal{R}^5 \rangle_{\alpha-\text{subleading}} \propto \frac{1}{\alpha} \frac{1}{k_1^3 k^9}$$

Subleading by $1/\alpha$ but enhanced by $k/k_1$ which for CMB can be as large as $10^3$

Leblond, NG2011, MCTP

2nd Problem

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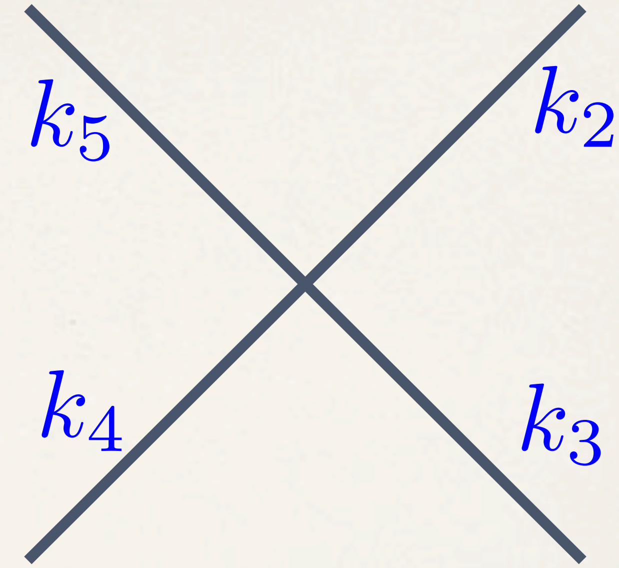


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Leblond, NG2011, MCTP

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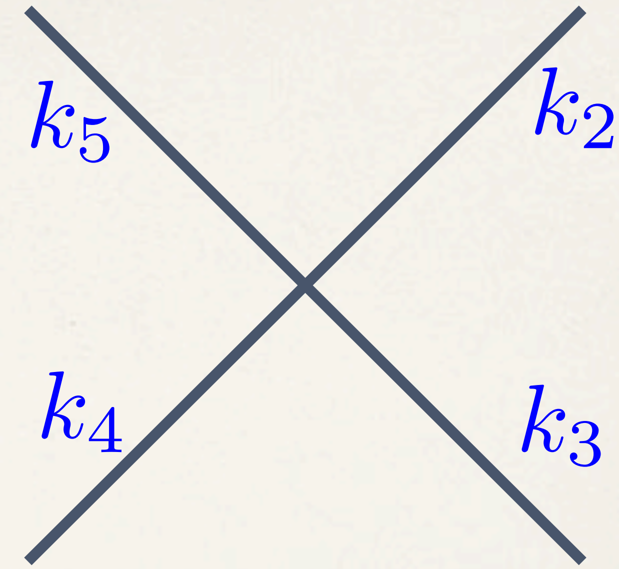
$\propto$

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Leblond, NG2011, MCTP

## Consistency relations

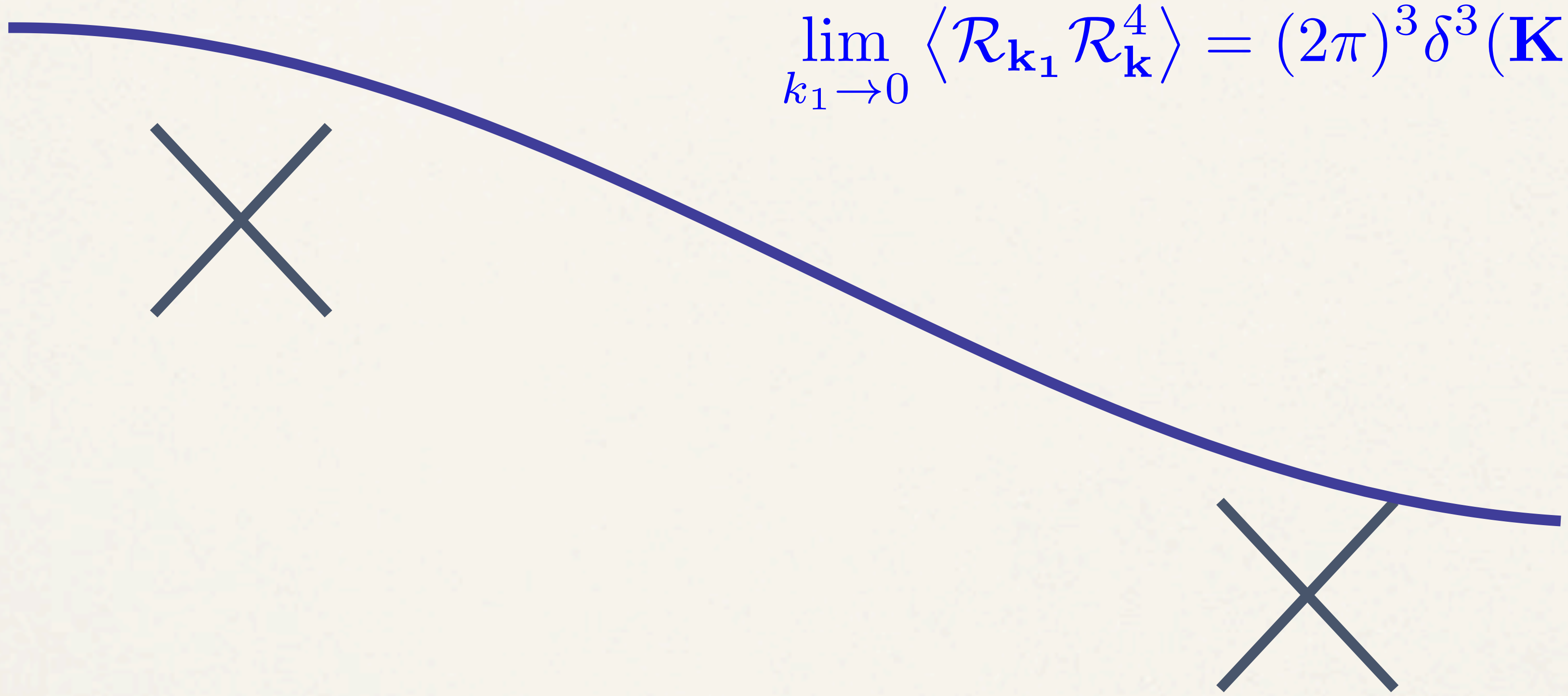
Maldacena
Creminelli, Zaldarriaga

more recently
Ganc, Komatsu

The squeezed mode acts as background modulating the lower point function

$$\left\langle \prod_{i=1}^{N} \mathcal{R}_{\mathbf{k}_i} \right\rangle_{\mathcal{R}^B} = \left\langle \prod_{i=1}^{N} \mathcal{R}_{\mathbf{k}_i} \right\rangle_{\mathcal{R}_B=0} + \mathcal{R}_B \left[ \frac{\partial}{\partial \mathcal{R}^B} \left\langle \prod_{i=1}^{N} \mathcal{R}_{\mathbf{k}_i} \right\rangle_{\mathcal{R}^B} \right]_{\mathcal{R}^B=0} + \mathcal{O}\left(\mathcal{R}_B^2\right)$$

$$\lim_{k_1 \to 0} \langle \mathcal{R}_{\mathbf{k_1}} \mathcal{R}_{\mathbf{k}}^4 \rangle = (2\pi)^3 \delta^3(\mathbf{K}) \sqrt{2\epsilon} |\mathcal{R}_{k_1}|^2 \frac{\partial}{\partial \phi_*} \langle R_{\mathbf{k}}^4 \rangle$$



Main message: we can calculate the shape and amplitude around squeezed limit using this consistency relation and our α-leading results.

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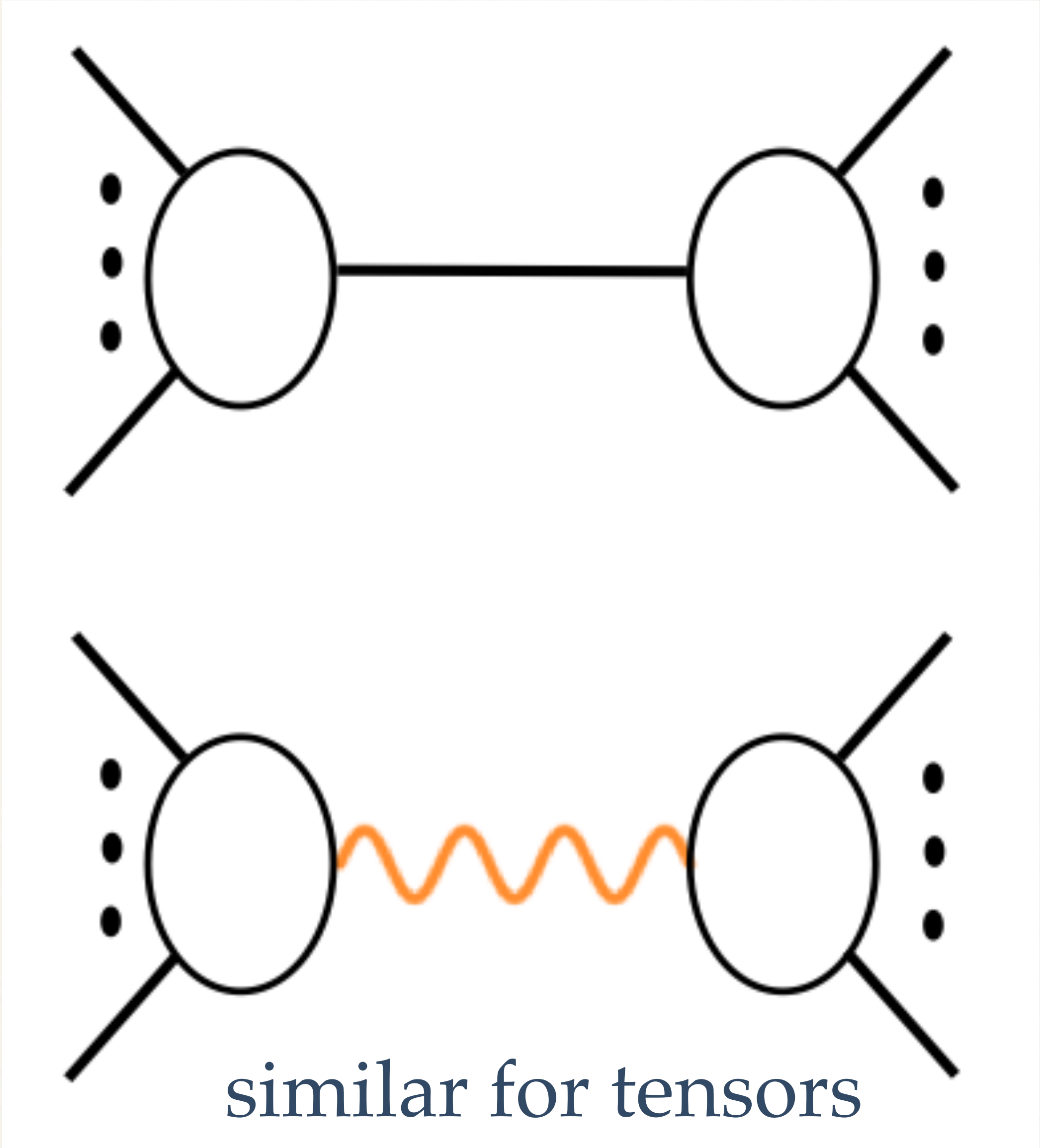
# Collinear Limits

$\vec{k}_1 + \vec{k}_2 \to 0$

$$\lim_{q\to 0} \left\langle \prod_{i=1}^{N} \mathcal{R}_{\mathbf{k}_i} \right\rangle^{SE} = \left\langle \left\langle \prod_{i=1}^{r} \mathcal{R}_{\mathbf{k}_i} \right\rangle_{\mathcal{R}_q} \left\langle \prod_{i=r+1}^{N} \mathcal{R}_{\mathbf{k}_i} \right\rangle_{\mathcal{R}_{q'}} \right\rangle$$

$$= 2\epsilon \left\langle \mathcal{R}_{\mathbf{q}'} \mathcal{R}_{\mathbf{q}} \right\rangle' \frac{\partial}{\partial \phi_*} \left\langle \prod_{i=1}^{r} \mathcal{R}_{\mathbf{k}_i} \right\rangle \frac{\partial}{\partial \phi_*} \left\langle \prod_{i=r+1}^{N} \mathcal{R}_{\mathbf{k}_i} \right\rangle$$

$$\frac{k^3}{|\vec{k}_1 + \vec{k}_2|^3} \sim 10^9$$




Using various consistency relations applied on the leading answers we can get the shape and amplitude at all squeezed/collinear limits.

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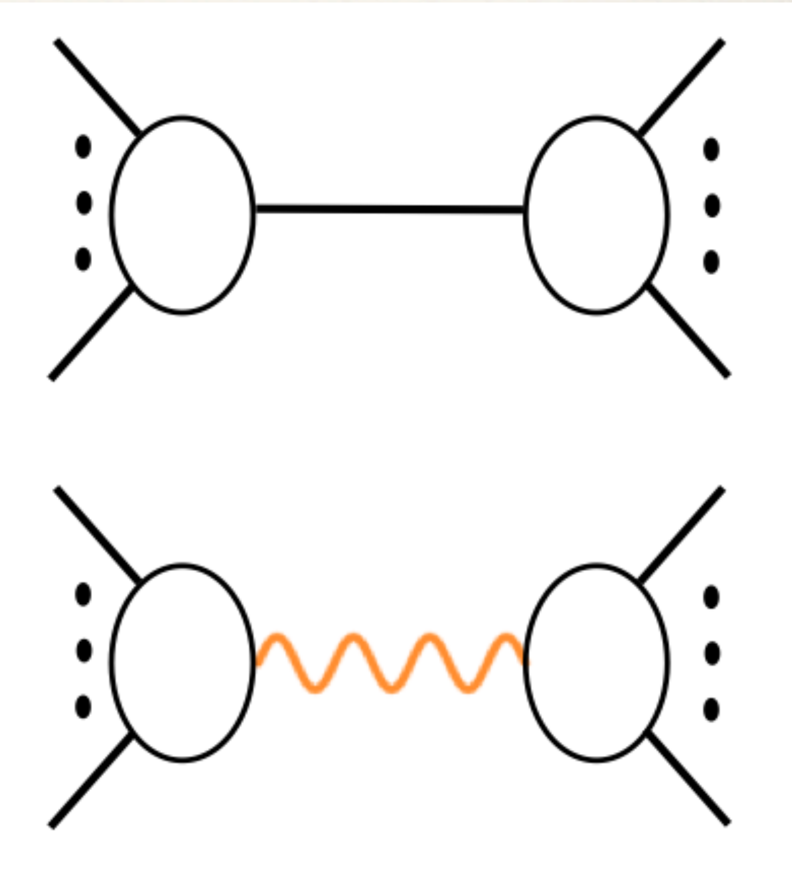
# Collinear limit of tensors

$$k^2 \to k^a k_a + k^a \gamma_{ab} k^b = k^a k_a - k_a \gamma^{ab} k_b$$

$$\lim_{q\to 0} \left\langle \prod_{i=1}^{N} \mathcal{R}_{\mathbf{k}_i} \right\rangle'^{GE} = |\gamma_q|^2 \sum_{\{i,j\}=1}^{N_1-1} \sum_{\{l,m\}=1}^{N_2-1} E_{ijlm} \frac{\partial}{\partial(\mathbf{k}_i \cdot \mathbf{k}_j)} \left\langle \prod_{i=1}^{N_1} \mathcal{R}_{\mathbf{k}_i} \right\rangle' \frac{\partial}{\partial(\mathbf{k}_l \cdot \mathbf{k}_m)} \left\langle \prod_{i=1}^{N_2} \mathcal{R}_{\mathbf{k}_i} \right\rangle'$$

$$E_{ijlm} = k_i k_j k_l k_m \sin\theta_i \sin\theta_j \sin\theta_l \sin\theta_m \cos(\phi_i + \phi_j - \phi_l - \phi_m)$$



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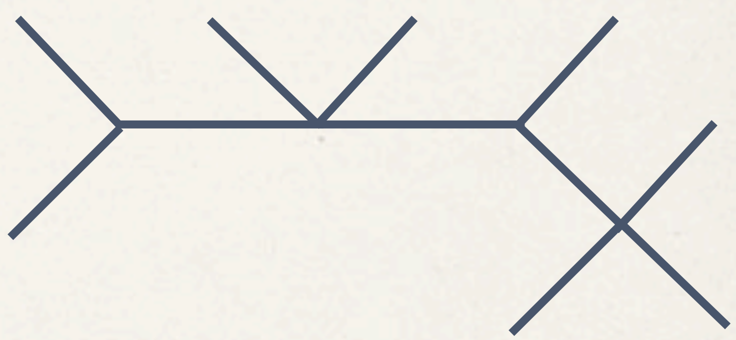
# Conclusions

- Decoupling limit is quite general and applies to any single field models with large NG.

- In resonant models of inflation we know higher N-point correlation functions that are valid for N up to 10 or more.

$$B_N(k_i) \equiv \frac{1}{K^{N-3}\prod_i k_i^2} \sin\left(\frac{\phi_K}{f}\right)$$

    - Main limit are from the proliferation of Feynman diagrams.



- Consistency relations can be used to predict the correct behaviour in all sorts of squeezed or collinear limits. Our results may provide a test case to study NG beyond the bispectrum.

# Trispectrum in collinear

$$\langle \mathcal{R}_{\mathbf{k}_1} \cdots \mathcal{R}_{\mathbf{k}_4} \rangle^{\text{collinear}} = (2\pi)^3 \delta^3 \left( \sum_{i=1}^{4} \mathbf{k}_i \right) \left\{ \frac{A_4}{K \prod_i k_i^2} \left[ \sin(a) - \frac{1}{\alpha} \cos(a) \sum_{j,i} \frac{k_i}{k_j} \right] \right.$$

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# Trispectrum in collinear

$$\langle \mathcal{R}_{\mathbf{k}_1} \cdots \mathcal{R}_{\mathbf{k}_4} \rangle^{\text{collinear}} = (2\pi)^3 \delta^3 \left( \sum_{i=1}^{4} \mathbf{k}_i \right) \left\{ \frac{A_4}{K \prod_i k_i^2} \left[ \sin(a) - \frac{1}{\alpha} \cos(a) \sum_{j,i} \frac{k_i}{k_j} \right] \right.$$

$$+ \frac{9}{16} r \sin^2 \theta_1 \sin^2 \theta_3 \cos\left[2(\phi_1 + \phi_3)\right] \frac{(2\pi^2 \Delta_R^2)^3}{k_{12}^3 k_1^3 k_3^3}$$

$$+ \frac{18\pi b^2 \alpha \sin^2(a) (2\pi^2 \Delta_R^2)^3}{k_{12}^3 k_1^3 k_3^3} + \text{perm}(\mathrm{k}_{13}, \mathrm{k}_{23})$$

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